Logaritmus

exponenciális függvény: alap , kitevő → hatvány

2 , 3 → $8 = 2^3$

logaritmus függvény: alap , hatvány → kitevő

2 , 8 → 3

kettes alapú logaritmus nyolc: $\log_2 8$

A kitevőt keressük.

1, $\log_2 32 = x$ $\quad x = 5$

2, $\log_3 81 = x$ $\quad x = 4$

3, $\log_2 \frac{1}{4} = x$ $\quad x = -2$

4, $\log_3 \frac{1}{27} = x$ $\quad x = -3$

5, $\log_{\frac{1}{2}} 8 = x$ $\quad x = -3$

6, $\log_{\frac{1}{3}} 9 = x$ $\quad x = -2$

7, $\log_{\frac{1}{2}} \frac{1}{16} = x$ $\quad x = 4$

8, $\log_{\frac{1}{5}} \frac{1}{125} = x$ $\quad x = 3$

1, $\log_2 x = 6$ $\quad x = 2^6 = 64$ $\quad$ A hatványt keressük

2, $\log_3 x = 1$ $\quad x = 3^1 = 3$

3, $\log_2 x = -4$ $\quad x = 2^{-4} = \frac{1}{16}$

4, $\log_4 x = -3$ $\quad x = 4^{-3} = \frac{1}{64}$

5, $\log_{\frac{1}{2}} x = 5$ $\quad x = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

6, $\log_{\frac{1}{3}} x = 3$ $\quad x = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

7, $\log_{\frac{1}{2}} x = -2$ $\quad x = \left(\frac{1}{2}\right)^{-2} = 4$

8, $\log_{\frac{1}{5}} x = -1$ $\quad x = \left(\frac{1}{5}\right)^{-1} = 5$

1, $\log_x 49 = 2$ $\quad x = 7$ $\quad x^2 = 49$ $\quad$ Az alapot keressük

2, $\log_x 16 = 4$ $\quad x = 2$

3, $\log_x 32 = -5$ $\quad x = \frac{1}{2}$

4, $\log_x 64 = -3$ $\quad x = \frac{1}{4}$

5, $\log_x \frac{1}{8} = 3$ $\quad x = \frac{1}{2}$

6, $\log_x \frac{1}{125} = 3$ $\quad x = \frac{1}{5}$

7, $\log_x \frac{1}{36} = -2$ $\quad x = 6$

8, $\log_x \frac{1}{128} = -7$ $\quad x = 2$

Definíció: $a, b > 0$ (csak pozitív szám lehet), $a \neq 1$ (nem lehet egy)

,a' alapú logaritmus b azt a kitevőt jelenti amire ,a'-t emelve b-t kapunk.

$$a^{\log_a b} = b \qquad a \neq 0$$

$$178^{\log_{178} 519} = 519$$

$$5^{\log_5 4+1} = \cancel{5^{\log_5}}4 \cdot 5^1 = 4 \cdot 5 = 20$$

$$2^{\log_2 7+3} = \cancel{2^{\log_2}}7 \cdot 2^3 = 7 \cdot 2^3 = 7 \cdot 8 = 56$$

$$3^{\log_3 5-2} = 1, \ 3^{\log_3 5} \cdot 3^{-2} = 5 \cdot \frac{1}{9} = \frac{5}{9}$$

$$2, \ \frac{\cancel{3^{\log_3}}5}{3^2} = \frac{5}{9}$$

$$4^{\log_2 7} = (2^2)^{\log_2 7} = \left(\cancel{2^{\log_2}}7\right)^2 = 7^2 = 49$$

$8^{\log_2 5} = (2^3)^{\log_2 5} = \left(\cancel{2}^{\cancel{\log_2} 5}\right)^3 = 5^3 = 125$

$36^{\log_6 8} = (6^2)^{\log_6 8} = \left(6^{\log_6 8}\right)^2 = 8^2 = 64$

$27^{\log_3 4} = (3^3)^{\log_3 4} = \left(\cancel{3}^{\cancel{\log_3} 4}\right)^3 = 4^3 = 64$

$16^{\log_4 7 + 2} = 16^{\log_4 7} \cdot 16^2 = (4^2)^{\log_4 7} \cdot 16^2 =$

$= \left(\cancel{4}^{\cancel{\log_4} 7}\right)^2 \cdot 16^2 = 7^2 \cdot 16^2 = 49 \cdot 256 = 12544$

A logaritmus azonosságai

$a, b, x > 0 \qquad a \neq 1$

I. $\log_a x + \log_a y = \log_a (x \cdot y)$

II. $\log_a x - \log_a y = \log_a \frac{x}{y}$

III. $\log_a x^n = n \cdot \log_a x$

$\lg x = \log_{10} x \qquad \log$

$\ln x = \log_e x \qquad e =$ természetes állandó

$\log_3 x + \log_3 5 = 1$ kikötés: $x > 0$

$\log_3 (x \cdot 5) = \log_3 3$ mivel a logaritmus függ. szigorúan monoton

mivel log függv. szig. mon.

$x \cdot 5 = 3 \quad /:5$

$x = \underline{\underline{\frac{3}{5}}}$

$\log_2 (x+3) - \log_2 (x-4) = 3$ kikötés: $x+3 > 0 \quad /-3$

$x > -3$

$x-4 > 0 \quad /-4$

$x > 4$

$\log_2 \frac{x+3}{x-4} = \log_2 8$

mivel a log függv. szig. mon.

$\frac{x+3}{x-4} = 8 \quad / \cdot x-4$

$x+3 = 8 \cdot (x-4)$

$x+3 = 8x - 32 \quad /-x$

$3 = 7x \quad /+32$

$35 = 7x \quad /:7$

$\underline{\underline{5}} = x$

$\log_7 (x-1) + \log_7 (x+2) = 2 \cdot \log_7 (x-3)$ kikötés

$\log_7 (x-1) \cdot (x+2) = \log_7 (x-3)^2$

mivel log. függv. szig. mon.

$(x-1) \cdot (x+2) = (x-3)^2$

$x^2 + 2x - x - 2 = x^2 - 6x + 9 \quad /-x^2$

$x - 2 = -6x + 9 \quad /+6x$

$7x - 2 = 9 \quad /+2$

$7x = 11 \quad /:7$

$x = \frac{11}{7}$

nincs megoldás

$x-1 > 0$

$x > 1$

$x+2 > 0$

$x > -2$

$x-3 > 0$

$x > 3$

$\log_3 (x-2) + \log_3 5 = 2$

$\log_3 x = 2$

kikötés $x - 2 > 0$

$x > 2$

$\log_3 (x-2)\cdot 5 = \log_3 9$

mivel a log fügv. szig. mon

$(x-2)\cdot 5 = 9$

$5x - 10 = 9 \quad /+10$

$5x = 19 \quad /:5$

$x = \frac{19}{5} = 3{,}8$

$\log_4 (x+3) - \log_4 (x-1) = 1$

kikötés $x + 3 > 0$

$x > -3$

$x - 1 > 0$

$x > 1$

$\log_4 \frac{x+3}{x-1} = \log_4 4$

mivel a log fügv. szig. mon

$\frac{x+3}{x-1} = 4 \quad /\cdot x-1$

$x + 3 = 4\cdot(x-1)$

$x + 3 = 4x - 4 \quad /-x$

$3 = 3x \quad /+4$

$7 = 3x \quad /:3$

$\frac{7}{3} = x$

$\log_5 (x+3) + \log_5 (x-5) = 2\cdot \log_5 (x+2)$

kikötés

$x + 3 > 0$

$x > -3$

$x - 5 > 0$

$x > 5$

$x + 2 > 0$

$x > -2$

$\log_5 (x+3)\cdot(x-5) = \log_5 (x+2)^2$

$(x+3)\cdot(x-5) = (x+2)^2$

$x^2 - 5x + 3x - 15 = x^2 + 4x + 4 \quad /-x^2$

$-2x - 15 = 4x + 4 \quad /+2x$

$-15 = 6x \quad /-4$

$-19 = 6x \quad /:6$

$-\frac{19}{6} = x$ ↯

$\log_4 7 + \log_4 (x-5) = 2$

kikötés

$x - 5 > 0$

$x > 5$

$\log_4 7 \cdot (x-5) = \log_4 16$

$7 \cdot (x-5) = 16$

$7x - 35 = 16 \quad / +35$

$7x = 51 \quad / :7$

$x = \frac{51}{7}$

$\log_3 (x-2) - \log_3 (x+7) = 3$

kikötés

$x - 2 > 0$

$x > 2$

$x + 7 > 0$

$x > -7$

$\log_3 \frac{x-2}{x+7} = \log_3 27$

mivel a log. fgv. szig. mon.

$\frac{x-2}{x+7} = 27 \quad / \cdot x+7$

$x - 2 = 27 \cdot (x+7)$

$x - 2 = 27x + 189 \quad / -x$

$-2 = 26x \quad / -189$

$-191 = 26x \quad / :26$

$-\frac{191}{26} = x$ h

$\log_{11} (x-8) + \log_{11} (x+7) = 2 \cdot \log_{11} (x+6)$

kikötés

$x - 8 > 0$

$x > 8$

$x + 7 > 0$

$x > -7$

$x + 6 > 0$

$x > -6$

$\log_{11} (x-8) \cdot (x+7) = \log_{11} (x+6)^2$

$(x-8) \cdot (x+7) = (x+6)^2$

$x^2 + 7x - 8x - 56 = x^2 + 36 + 12x \quad / -x^2$

$-x - 56 = 36 + 12x \quad / +x$

$-56 = 36 + 13x \quad / -36$

$-92 = 13x \quad / :13$

$\frac{-92}{13} = x$ h

$\lg(x-1) + \lg 4 = 2$ $\qquad$ $x - 1 > 0$

$\log_{10}(x-1) + \log_{10} 4 = \log_{10} 100$ $\qquad$ $x > 1$

mivel a log. fügv. szig. mon.

$\log_{10}(x-1)\cdot 4 = \log_{10} 100$

$4\cdot(x-1) = 100$

$4x - 4 = 100 \quad / + 4$

$4x = 104 \quad / : 4$

$x = \underline{\underline{26}}$

7 p.

kikötés

$\log_3\left(\sqrt{x+1} + 1\right) = 2$ $\qquad$ $x \geq -1$

$\log_3\left(\sqrt{x+1} + 1\right) = \log_3 9$

mivel a log függv. szig. mon.

$\sqrt{x+1} + 1 = 9 \quad / - 1$

$\sqrt{x+1} = 8 \quad / ()^2$

$x + 1 = 64 \quad / - 1$

$x = \underline{\underline{63}}$

6 p.

$\log_{10} x = \log_{10} 3 + \log_{10} 25$

$\log_{10} x = \log_{10}(3\cdot 25)$

2 p.

$x = \underline{\underline{75}}$

kikötés

$2 \cdot \log_{10}(y+1) = \log_{10}(x+11)$

$y+1 > 0$

$y = 2x$

$y > -1$

$\log_{10}(2x+1)^2 = \log_{10}(x+11)$

$x+11 > 0$

$x > -11$

mivel a log fív. szig. mon.

$(2x+1)^2 = x+11$

$4x^2+1+4x = x+11 \quad / -x$

$4x^2+1+3x = 11 \quad / -11$

$4x^2-10+3x = 0 \qquad a=4 \quad b=3 \quad c=-10$

$x_{1,2} = \frac{-3 \pm \sqrt{\overset{9}{3^2} - \overset{+160}{4 \cdot 4 \cdot (-10)}}}{2 \cdot 4} = \frac{-3 \pm \sqrt{169}}{8} = \frac{-3 \pm 13}{8}$

$x_1 = \frac{-3+13}{8} = \frac{10}{8} = \underline{\underline{1{,}25}} \qquad y_1 = \underline{\underline{2{,}5}}$

11p

$x_2 = \frac{-3-13}{8} = \frac{-16}{8} = -2 \qquad y_2 = -4$ ↯

$\lg x^2 = 2 \cdot \lg x$

kikötés

$x > 0$

$\lg x^2 = \lg x^2$

mivel a log fív. szig. mon.

$x = \underline{\underline{\in R^+}}$

$x^2 = x^2$

$x = \in R$

2p

$$\lg (x+15)^2 - \lg (3x+5) = \lg 20$$

kikötés:

$3x + 5 > 0$

$3x > -5$

$x > -\frac{5}{3}$

$x + 15 \neq 0$

$x \neq 15$

$$\lg \frac{(x+15)^2}{3x+5} = \lg 20$$

mivel a log fűv. szig mon

$$\frac{(x+15)^2}{3x+5} = 20 \qquad / \cdot 3x+5$$

$$(x+15)^2 = 60x + 100$$

$$x^2 + 225 + 30x = 60x + 100 \qquad / -100$$

$$x^2 + 125 + 30x = 60x \qquad / -60x$$

$$x^2 + 125 - 30x = 0$$

$$x_{1,2} = \frac{30 \pm \sqrt{\overset{900}{-30^2} - \overset{-500}{4 \cdot 1 \cdot 125}}}{2 \cdot 1} = \frac{30 \pm \sqrt{400}}{2} = \frac{30 \pm 20}{2}$$

$$x_1 = \frac{30+20}{2} = \frac{50}{2} = \underline{\underline{25}}$$

6 p.

$$x_2 = \frac{30-20}{2} = \frac{10}{2} = \underline{\underline{5}}$$

kikötés

$x > 0$

$(\log_2 x - 3) \cdot (\log_2 x^2 + 6) = 0$

$\log_2 x - 3 = 0 \quad /+3$

$\log_2 x = 3$

$\underline{\underline{x = 8}}$

$\log_2 x^2 + 6 = 0 \quad /-6$

$\log_2 x^2 = -6$

$2 \cdot \log_2 x = -6 \quad /:2$

$\log_2 x = -3$

$x = 2^{-3} = \underline{\underline{\frac{1}{8}}}$

$\lg p_m = 0{,}8 \cdot \lg p_v + 0{,}301$

a, mennyit mér 20 Pa valódi nyomásnál?

b, mennyi a valódi nyomás, ha 50 Pa-t mér?

c, mekkora nyomás esetén mutatja a valódi nyomást?

a, $\lg p_m = 0{,}8 \cdot \lg 20 + 0{,}301$

$\lg p_m = 1{,}3418$

$p_m = 10^{1{,}3418} = 21{,}97 \approx 22$ Pa

b, $\lg 50 = 0{,}8 \cdot \lg p_v + 0{,}301 \quad /-0{,}301$

$\lg 50 - 0{,}301 = 0{,}8 \cdot \lg p_v \quad /:0{,}8$

$\frac{\lg 50 - 0{,}301}{0{,}8} = \lg p_v$

$1{,}7474 = \lg p_v$

$\lg p_v = 10^{1{,}7474} = 55{,}89 \approx 56$

c, $p_m = p_v$

$\lg p_v = 0{,}8 \cdot \lg p_v + 0{,}301 \quad /-0{,}8 \lg p_v$

$0{,}2 \lg p_v = 0{,}301 \quad /:0{,}2$

$\lg p_v = 1{,}505$

$\lg p_v = 10^{1{,}505} = 31{,}98 \approx 32$

5 ember

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ össz leülési lehetőség

A B, C D E

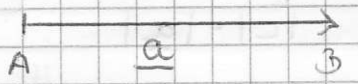
$4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 = 48$ részleges

$120 - 48 = \underline{\underline{72}}$ féle képpen ülhetnek le

Vektorok

(irányított szakasz)

jelölése: $\overrightarrow{AB} = \underline{a}$



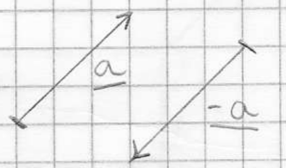
Vektor hossza (abszolútértéke) $|\overrightarrow{AB}|$

Nullvektor: - hossza: nulla
- iránya: tetszőleges

jele: $\underline{0}$

Vektor ellentetje: Ugyanolyan nagyságú, ellentétes irányú vektor.

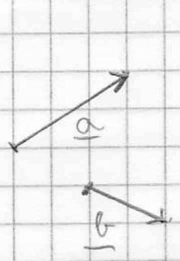
pl.:

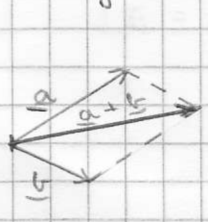


Vektor műveletek:

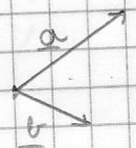
1, összeadás

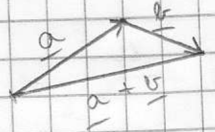
paralelogramma módszer





háromszög módszer





2, különbség (kivonás)

$\underline{a} - \underline{b} = \underline{a} + (-\underline{b})$

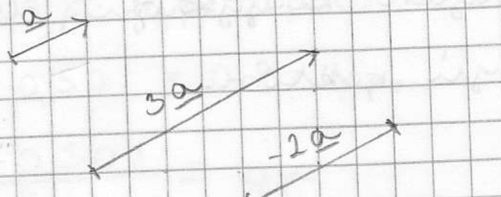
3, számmal (skalárral) való szorzás

$c \in \mathbb{R}$

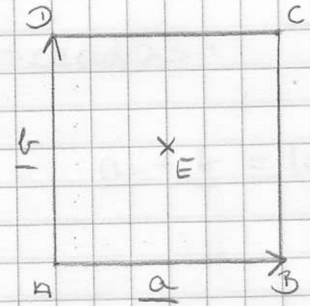
$\underline{a}$

$c \cdot \underline{a}$
- hossza: $|c| \cdot |\underline{a}|$
- iránya:
  - ha $c > 0$, akkor a-val egyirányú
  - ha $c < 0$, akkor a-val ellentétes irányú

pl.:



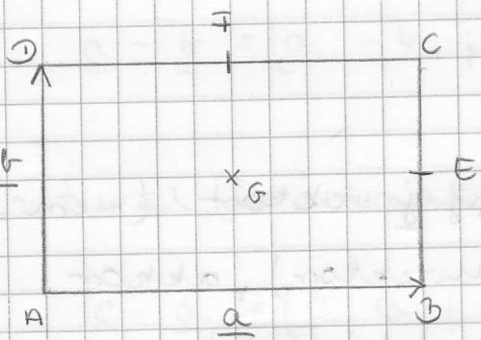
1,



$\overrightarrow{DB} = \underline{a} - \underline{b}$

$\overrightarrow{AE} = \dfrac{\underline{a} + \underline{b}}{2}$

2,



$\overrightarrow{DE} = \underline{a} - \dfrac{\underline{b}}{2}$

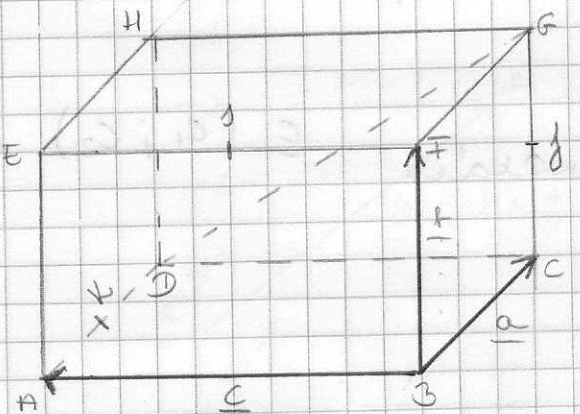
$\overrightarrow{BF} = \underline{b} + -\dfrac{\underline{a}}{2}$

$\overrightarrow{GE} = \frac{1}{2}\underline{a}$

$\overrightarrow{CG} = -\frac{1}{2}\underline{a} - \frac{1}{2}\underline{b}$

$\overrightarrow{AF} = \underline{b} + \frac{1}{2}\underline{a}$

3,



$\overrightarrow{DG} = \underline{b} - \underline{c}$

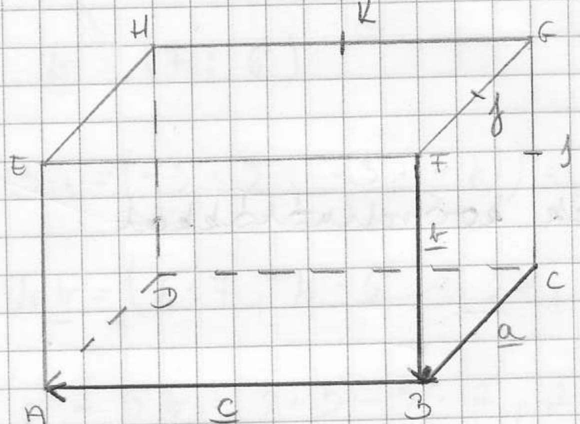
$\overrightarrow{EJ} = -\underline{c} + \underline{a} - \frac{1}{2}\underline{b}$

$\overrightarrow{IC} = -\frac{1}{2}\underline{c} - \underline{b} + \underline{a}$

$\overrightarrow{KF} = -\frac{1}{2}\underline{a} - \underline{c} + \underline{b}$

$\overrightarrow{IK} = \frac{1}{2}\underline{c} - \underline{b} + \frac{1}{2}\underline{a}$

4,



$\overrightarrow{AI} = -\underline{c} - \underline{a} - \frac{1}{2}\underline{b}$

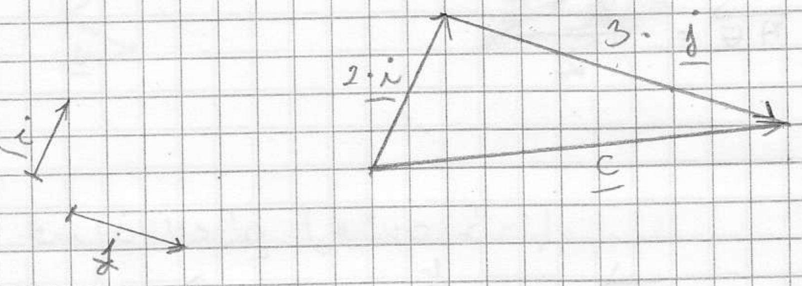
$\overrightarrow{DJ} = -\underline{c} - \underline{b} + \frac{1}{2}a$

$\overrightarrow{KB} = -\frac{1}{2}\underline{c} + \underline{a} + \underline{b}$

$\overrightarrow{JA} = \frac{1}{2}\underline{a} + \underline{b} + \underline{c}$

$\overrightarrow{IE} = -\frac{1}{2}\underline{b} + \underline{a} + \underline{c}$

Vektorok koordinátái



Ha adott a síkon egy $\underline{i}$ és egy $\underline{j}$ vektor (nem párhuzamosak és egyik sem nullvektor), akkor tetszőleges vektora egyértelműen felírható

$\underline{c} = c_1 \cdot \underline{i} + c_2 \cdot \underline{j}$ $\quad (c_1 ; c_2 \in \mathbb{R})$ alakban.

$\underline{i}$; $\underline{j}$: a sík bázisvektorai $\qquad \underline{c}\ (c_1 ; c_2)$

$c_1 ; c_2$: a $\underline{c}$ vektor koordinátái.

1, $\underline{a} = 4\underline{i} - 3\underline{j}$

$\underline{b} = 2\underline{i} + 5\underline{j}$

$3\underline{a} + 2\underline{b} = 3 \cdot (4\underline{i} - 3\underline{j}) + 2 \cdot (2\underline{i} + 5\underline{j}) = 12\underline{i} - 9\underline{j} + 4\underline{i} + 10\underline{j}$

$= 16\underline{i} + \underline{j}$

Vektorműveletek koordinátákkal

$\underline{a}\ (a_1 ; a_2)$

$\underline{b}\ (b_1 ; b_2)$

$c \in \mathbb{R}$

összeadás:

$$\underline{a}+\underline{b}=(a_1+b_1\,;\,a_2+b_2)$$

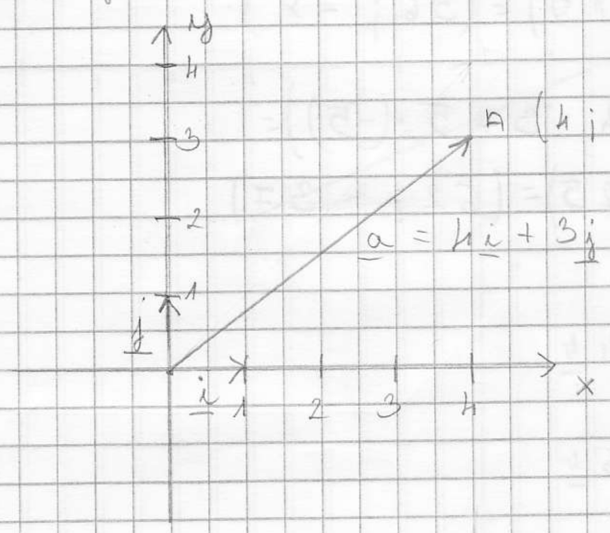
kivonás:

$$\underline{a}-\underline{b}=(a_1-b_1\,;\,a_2-b_2)$$

skalárral (számmal) való szorzás

$$c\cdot\underline{a}=(c\cdot a_1\,;\,c\cdot a_2)$$

Derékszögű koordináta rendszer



Helyvektor:

Origókezdőpontú vektor

A helyvektor koordinátái megegyeznek a végpontjának koordinátáival.

2, $\underline{a}\ (5;-4)$

$\underline{b}\ (7;6)$

$$-3a=(-3\cdot5\,;\,-3\cdot(-4))=(-15\,;\,12)$$

$$4\underline{b}=(4\cdot7\,;\,4\cdot6)=(28\,;\,24)$$

$$2\underline{a}-5\underline{b}=(2\cdot5-5\cdot7\,;\,2\cdot(-4)-5\cdot6)=(10-35\,;\,-8-30)$$
$$=(-25\,;\,-38)$$

3. $\underline{a}\ (-8;3)$

$\underline{b}\ (6;-5)$

$3\underline{a}-4\underline{b}=(3\cdot(-8)-4\cdot 6;\ 3\cdot 3-4\cdot(-5))=(-24-24;\ 9+20)=$
$=(-48;\ 29)$

$5\underline{a}+3\underline{b}=(5\cdot(-8)+3\cdot 6;\ 5\cdot 3+3\cdot(-5))=(-40+18;\ 15-15)$
$=(-22;\ 0)$

$-6\underline{a}-2\underline{b}=(-6\cdot(-8)-2\cdot 6;\ -6\cdot 3;\ -2\cdot(-5))=$
$=(48-12;\ -18+10)=(36;\ -8)$

$-4\underline{a}+5\underline{b}=(-4\cdot(-8)+5\cdot 6;\ -4\cdot 3+5\cdot(-5))=$
$=(32+30;\ -12-25)=(62;\ -37)$

Hf.: $\underline{a}\ (-9;7)$ $\quad 3\underline{a}+4\underline{b}$

$\underline{b}\ (11;4)$ $\quad -7\underline{a}-6\underline{b}$

$2\underline{a}-5\underline{b}=(2\cdot(-9)-5\cdot 11;\ 2\cdot 7-5\cdot 4)=$
$=(-18-55;\ 14-20)=(-73;\ -6)$

$3\underline{a}+4\underline{b}=(3\cdot(-9)+4\cdot 11;\ 3\cdot 7+4\cdot 4)=$
$=(-27+44;\ 21+16)=(17;\ 37)$

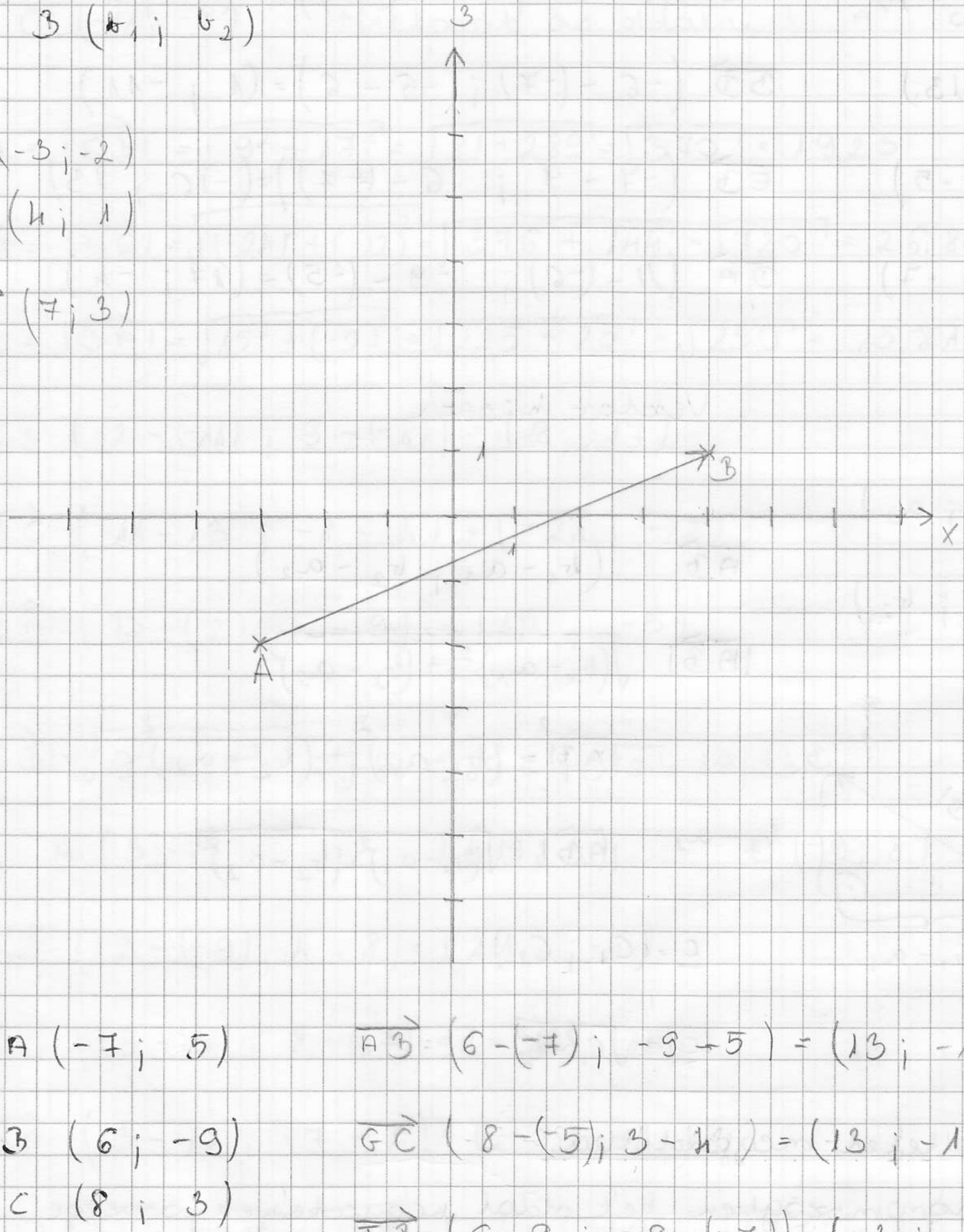
$-7\underline{a}-6\underline{b}=(-7\cdot(-9)-6\cdot 11;\ -7\cdot 7-6\cdot 4)=$
$=(63-66;\ -49-24)=(-3;\ -73)$

$\overrightarrow{AB}$ vektor koordinátái:

$A\,(a_1;\ a_2)$

$B\,(b_1;\ b_2)$

$\overrightarrow{AB}\,(b_1 - a_1;\ b_2 - a_2)$



$A\,(-3;\ -2)$

$B\,(4;\ 1)$

$\overrightarrow{AB}\,(7;\ 3)$

4,

$A\,(-7;\ 5)$ $\qquad \overrightarrow{AB}\,(6-(-7);\ -9-5) = (13;\ -14)$

$B\,(6;\ -9)$ $\qquad \overrightarrow{GC}\,(8-(-5);\ 3-4) = (13;\ -1)$

$C\,(8;\ 3)$

$D\,(-4;\ -8)$ $\qquad \overrightarrow{FB}\,(6-9;\ -9-(-7)) = (-3;\ -2)$

$E\,(11;\ 5)$ $\qquad \overrightarrow{DE}\,(11-(-4);\ 5-(-8)) = (15;\ 13)$

$F\,(9;\ -7)$ $\qquad \overrightarrow{CF}\,(9-8;\ -7-3) = (1;\ -10)$

$G\,(-5;\ 4)$ $\qquad \overrightarrow{BE}\,(11-6;\ 5-(-9)) = (5;\ 14)$

$\overrightarrow{EA}\,(-7-11;\ 5-5) = (-18;\ 0)$

Hf.:

$A\ (11;\ -9)$ $\quad \overrightarrow{DE}\ \left(9-(-6);\ (-7)-(-5)\right) = (15;\ -2)$

$B\ (-7;\ 6)$ $\quad \overrightarrow{AC}\ \left(8-11;\ 13-(-9)\right) = (-3;\ 22)$

$C\ (8;\ 13)$ $\quad \overrightarrow{BD}\ \left(-6-(-7);\ -5-6\right) = (1;\ -11)$

$D\ (-6;\ -5)$ $\quad \overrightarrow{EB}\ \left(-7-9;\ 6-(-7)\right) = (-16;\ 13)$

$E\ (9;\ -7)$ $\quad \overrightarrow{DA}\ \left(11-(-6);\ -9-(-5)\right) = (17;\ -4)$

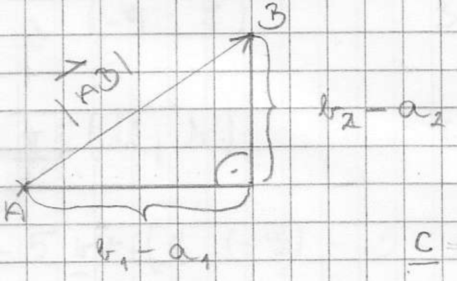
Vektor hossza

$A\ (a_1;\ a_2)$

$B\ (b_1;\ b_2)$

$\overrightarrow{AB}\ (b_1-a_1;\ b_2-a_2)$

$|\overrightarrow{AB}|\ \sqrt{(b_1-a_1)^2+(b_2-a_2)^2}$



$$|\overrightarrow{AB}|^2 = (b_1-a_1)^2+(b_2-a_2)^2$$

$$|\overrightarrow{AB}| = \sqrt{(b_1-a_1)^2+(b_2-a_2)^2}$$

$$\underline{c}\ (c_1;\ c_2)$$

$$|\underline{c}| = \sqrt{c_1^2+c_2^2}$$

Pitagorasz tétel megforditva:

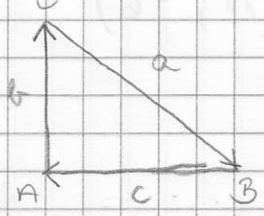
Ha egy háromszögben két oldal négyzetének öszege egyenlő a harmadik oldal négyzetével, akkor a háromszög derékszögű.

1, Egy háromszög csúcsai:

A $(13; 4)$

B $(-2; 9)$

C $(-11; -8)$ Mekkorák az oldalai?



$a = |\overrightarrow{CB}| = \sqrt{9^2 + 17^2} = \sqrt{81 + 289} = \sqrt{370} = 19,23$

$b = |\overrightarrow{AC}| = \sqrt{(-24)^2 + (-12)^2} = \sqrt{576 + 144} = \sqrt{720} = 26,83$

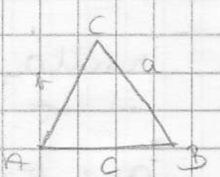
$c = |\overrightarrow{BA}| = \sqrt{15^2 + (-5)^2} = \sqrt{225 + 25} = \sqrt{250} = 15,81$

$\overrightarrow{CB}\ (-2-(-11);\ 9-(-8)) = (9;\ 17)$

$\overrightarrow{AC}\ (-11-13;\ -8-4) = (-24;\ -12)$

$\overrightarrow{BA}\ (13-(-2);\ 4-9) = (15;\ -5)$

$|\overrightarrow{CB}| = \sqrt{9^2 + 17^2} = \sqrt{81 + 289} = \sqrt{370} = 19,23$



2, A $(3; -7)$ B $(12; 4)$ C $(-9; 8)$

$\overrightarrow{CB}\ (12-(-9);\ 4-8) = (21;\ -4)$

$\overrightarrow{AC}\ (-9-3;\ 8-(-7)) = (-12;\ 15)$

$\overrightarrow{BA}\ (3-12;\ -7-4) = (-9;\ -11)$

$a = |\overrightarrow{CB}| = \sqrt{21^2 + (-4)^2} = \sqrt{441 + 16} = \sqrt{457} = 21,37$

$b = |\overrightarrow{AC}| = \sqrt{(-12)^2 + 15^2} = \sqrt{144 + 225} = \sqrt{369} = 19,21$

$c = |\overrightarrow{BA}| = \sqrt{(-9)^2 + (-11)^2} = \sqrt{81 + 121} = \sqrt{202} = 14,21$

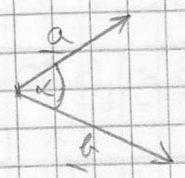
Hf:

A $(11; -6)$ B $(-2; 8)$ C $(-13; 4)$

Skaláris szorzat

$\underline{a}\ (a_1 ; a_2)$

$\underline{b}\ (b_1 ; b_2)$



$\underline{a} \cdot \underline{b} = |\underline{a}| \cdot |\underline{b}| \cdot \cos\alpha$

Ha $\underline{a} \cdot \underline{b} = 0$ akkor a két vektor merőleges egymásra. $\underline{a} \perp \underline{b}$

Ha $\underline{a} \perp \underline{b}$ akkor a két vektor szorzata 0
merőleges

Két vektor skaláris szorzat akkor és csak akkor nulla, ha merőleges a két vektor egymásra.

$\underline{a} \cdot \underline{b} = a_1 \cdot b_1 + a_2 \cdot b_2$

Határozzuk meg az $\underline{a}(5; -6)$ és a $\underline{b}(8; 7)$ vektorok által bezárt szöget.

$\underline{a} \cdot \underline{b}$

// \\

$a_1 \cdot b_1 + a_2 \cdot b_2 \quad |\underline{a}| \cdot |\underline{b}| \cdot \cos\alpha$

$|a| = \sqrt{5^2 + (-6)^2} = \sqrt{25 + 36} = \sqrt{61} = 7{,}81$

$|b| = \sqrt{8^2 + 7^2} = \sqrt{64 + 49} = \sqrt{113} = 10{,}63$

$a_1 \cdot b_1 + a_2 \cdot b_2 = |\underline{a}| \cdot |\underline{b}| \cdot \cos\alpha$

$5 \cdot 8 + -6 \cdot 7 = 7,81 \cdot 10,63 \cdot \cos\alpha$

$40 - 42 = 83,0203 \cdot \cos\alpha$

$-2 = 83,0203 \cdot \cos\alpha \qquad / : 83,0203$

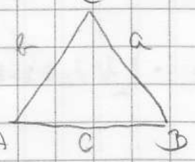
$\frac{-2}{83,0203} = \cos\alpha$

$91,38^\circ = \alpha$

A $(11 ; -6)$

B $(-2 ; 8)$

C $(-13 ; 4)$



$\overrightarrow{AB} \quad (-2 - 11 ; 8 - (-6)) = (-13 ; 14)$

$\overrightarrow{BC} \quad (-13 - (-2) ; 4 - 8) = (-11 ; -4)$

$\overrightarrow{CA} \quad (11 - (-13) ; -6 - 4) = (24 ; -10)$

$c = |\overrightarrow{AB}| = \sqrt{(-13)^2 + 14^2} = \sqrt{169 + 196} = = \sqrt{365} = 19,10$

$b = |\overrightarrow{CA}| = \sqrt{24^2 + (-10)^2} = \sqrt{576 + 100} = \sqrt{676} = 26$

$a = |\overrightarrow{BC}| = \sqrt{(-11)^2 + (-4)^2} = \sqrt{121 + 16} = \sqrt{137} = 11,7$

1, $\underline{a}$ $(-11;7)$ $|\underline{a}| = \sqrt{a_1^2 + a_2^2}$

$\underline{b}$ $(-9;8)$

$\underline{a} \cdot \underline{b}$

$\parallel$

$a_1 \cdot b_1 + a_2 \cdot b_2 = |\underline{a}| \cdot |\underline{b}| \cdot \cos\alpha$

$|\underline{a}| = \sqrt{(-11)^2 + 7^2} = \sqrt{121 + 49} = \sqrt{170} = 13{,}04$

$|\underline{b}| = \sqrt{(-9)^2 + 8^2} = \sqrt{81 + 64} = \sqrt{145} = 12{,}04$

$a_1 \cdot b_1 + a_2 \cdot b_2 = |\underline{a}| \cdot |\underline{b}| \cdot \cos\alpha$

$-11 \cdot -9 + 7 \cdot 8 = 13{,}04 \cdot 12{,}04 \cdot \cos\alpha$

$99 + 56 = 157{,} \cdot \cos\alpha \quad /: 157$

$\frac{155}{157} = \cos\alpha$

$9{,}15^\circ = \alpha$

2, A $(5;-7)$ $\overrightarrow{AB}$ és $\overrightarrow{CD}$ szöge

B $(8;9)$

C $(-6;3)$

D $(-7;-8)$

$\overrightarrow{AB}$ $(8-5;\ 9-(-7)) = (3;16)$

$\overrightarrow{CD}$ $(-7-(-6);\ -8-3) = (-1;-11)$

$\underline{a}$ $(3;16)$ $\underline{b}$ $(-1;-11)$

$|\vec{AB}| = \sqrt{3^2 + 16^2} = \sqrt{9 + 256} = \sqrt{265} = 16{,}28$

$|\vec{CD}| = \sqrt{(-1)^2 + (-11)^2} = \sqrt{1 + 121} = \sqrt{122} = 11{,}05$

$a_1 \cdot b_1 + a_2 \cdot b_2 = |\underline{a}| \cdot |\underline{b}| \cdot \cos\alpha$

$3 \cdot -1 + 16 \cdot -11 = 16{,}28 \cdot 11{,}05 \cdot \cos\alpha$

$-3 + -176 = 179{,}894 \cdot \cos\alpha \qquad /:179{,}89$

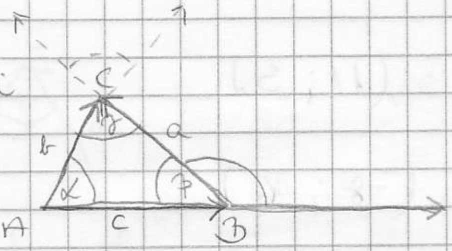
$\dfrac{-179}{179{,}89} = \cos\alpha$

$174{,}29^\circ = \alpha$

3, A $(9;3)$ oldalai szögei

B $(-2;8)$

C $(-7;-5)$



$\vec{AB} \quad (-2-9;\ 8-3) = (-11;\ 5)$

$\vec{BC} \quad (-7-(-2);\ -5-8) = (-5;\ -13)$

$\vec{AC} \quad (-7-9;\ -5-3) = (-16;\ -8)$

$c = |\vec{AB}| = \sqrt{(-11)^2 + 5^2} = \sqrt{121 + 25} = \sqrt{146} = 12{,}08$

$a = |\vec{BC}| = \sqrt{(-5)^2 + (-13)^2} = \sqrt{25 + 169} = \sqrt{194} = 13{,}93$

$b = |\vec{AC}| = \sqrt{(-16)^2 + (-8)^2} = \sqrt{256 + 64} = \sqrt{320} = 17{,}89$

$\alpha = \vec{AB} \cdot \vec{AC}$

$(-11) \cdot (-16) + 5 \cdot (-8) = 12{,}08 \cdot 17{,}89 \cdot \cos\alpha$

$176 - 40 = 216{,}11 \cdot \cos\alpha \qquad /:216{,}11$

$\dfrac{136}{216{,}11} = \cos\alpha$

$51^\circ = \alpha$

$\gamma = \overrightarrow{AC} \cdot \overrightarrow{BC}$

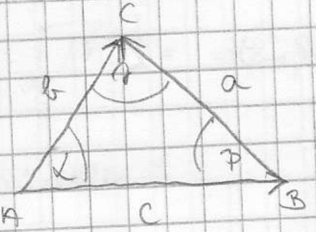
$(-16) \cdot (-5) + (-8) \cdot (-13) = 17{,}89 \cdot 13{,}93 \cdot \cos \gamma$

$80 + 104 = 249{,}21 \cdot \cos \gamma \quad | : 249{,}21$

$\frac{184}{249{,}21} = \cos \gamma$

$42{,}41^\circ = \gamma$

$\beta = 86{,}59^\circ$



$A \ (4; -9)$

$B \ (11; 3)$

$C \ (-8; 6)$

$\overrightarrow{AC} \ (-8-4; \ 6-(-9)) = (-12; \ 15)$

$\overrightarrow{BC} \ (-8-11; \ 6-3) = (-19; \ 3)$

$\overrightarrow{AB} \ (11-4; \ 3-(-9)) = (7; \ 12)$

$c = |\overrightarrow{AB}| = \sqrt{7^2 + 12^2} = \sqrt{49 + 144} = \sqrt{193} = 13{,}89$

$b = |\overrightarrow{AC}| = \sqrt{(-12)^2 + 15^2} = \sqrt{144 + 225} = \sqrt{369} = 19{,}21$

$a = |\overrightarrow{BC}| = \sqrt{(-19)^2 + 3^2} = \sqrt{361 + 9} = \sqrt{370} = 19{,}24$

$\alpha = \overrightarrow{AC} \cdot \overrightarrow{AB}$

$7 \cdot (-12) + 12 \cdot 15 = 13{,}89 \cdot 19{,}21 \cdot \cos \alpha$

$-84 + 180 = 266{,}83 \cdot \cos \alpha \quad | : 266{,}83$

$\frac{96}{266{,}83} = \cos \alpha$

$68{,}91^\circ = \alpha$

$\gamma = \vec{AC} \cdot \vec{BC}$

$-12 \cdot (-19) + 15 \cdot 3 = 19{,}21 \cdot 19{,}24 \cdot \cos\gamma$

$228 + 45 = 369{,}6 \cdot \cos\gamma \quad /: 369{,}6$

$\frac{273}{369{,}6} = \cos\gamma$

$42{,}38^\circ = \gamma$

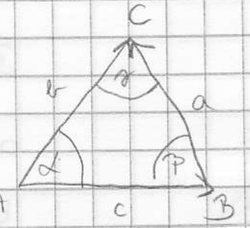
$\beta = 180 - 42{,}38 - 68{,}91 = 68{,}71^\circ$

Hf:

A $(-11; -9)$

B $(8; -3)$

C $(-4; 12)$



$\vec{AC}$ $(-4-(-11); 12-(-9)) = (7; 21)$

$\vec{AB}$ $(8-(-11); -3-(-9)) = (19; 6)$

$\vec{BC}$ $(-4-8; 12-(-3)) = (-12; 15)$

$b = |\vec{AC}| = \sqrt{7^2 + 21^2} = \sqrt{49 + 441} = \sqrt{490} = 22{,}14$

$a = |\vec{BC}| = \sqrt{(-12)^2 + 15^2} = \sqrt{144 + 225} = \sqrt{369} = 19{,}21$

$c = |\vec{AB}| = \sqrt{19^2 + 6^2} = \sqrt{361 + 36} = \sqrt{397} = 19{,}92$

$\alpha = \vec{AC} \cdot \vec{AB}$

$7 \cdot 19 + 21 \cdot 6 = 22{,}14 \cdot 19{,}92 \cdot \cos\alpha$

$133 + 126 = 441{,}03 \cdot \cos\alpha \quad /: 441{,}03$

$\frac{259}{441{,}03} = \cos\alpha$

$54{,}04^\circ = \alpha$

$\gamma = \overrightarrow{AC} \cdot \overrightarrow{BC}$

$7 \cdot (-12) + 21 \cdot 15 = 22{,}14 \cdot 19{,}21 \cdot \cos\gamma$

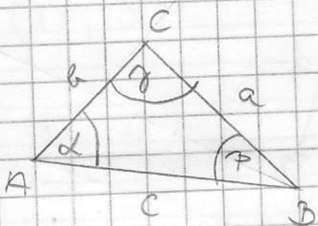
$-84 + 315 = 425{,}31 \cdot \cos\gamma \qquad / : 425{,}31$

$\frac{231}{425{,}31} = \cos\gamma$

$57{,}1^\circ = \gamma$

$\beta = 180 - 54{,}04 - 57{,}1 = 68{,}86^\circ$

Szinusztétel



Bármely háromszögben két oldal aránya egyenlő a szemközti szögek szinuszának arányával.

pl: $\frac{a}{b} = \frac{\sin\alpha}{\sin\beta}$ $\qquad$ $\frac{c}{a} = \frac{\sin\gamma}{\sin\alpha}$

- használható, ha adott
  - két szög és egy oldal
  - két oldal adott és a nagyobbal szemközti szög.

1, Egy háromszögben $\alpha = 39^\circ$, $\beta = 74^\circ$, $c = 21$ cm
Határozzuk meg a hiányzó adatokat.

$\gamma = 67^\circ \qquad \gamma = 180 - \alpha - \beta$

$\frac{a}{c} = \frac{\sin\alpha}{\sin\gamma} \qquad \frac{a}{21} = \frac{\sin 39^\circ}{\sin 67^\circ}$

$\frac{a}{21} = \frac{\sin 39^\circ}{\sin 67^\circ}$ $\quad / \cdot 21$

$a = \frac{21 \cdot \sin 39^\circ}{\sin 67^\circ}$

$a = 14,36$ cm

$\frac{b}{c} = \frac{\sin \beta}{\sin \gamma}$

$\frac{b}{21} = \frac{\sin 74^\circ}{\sin 67^\circ}$ $\quad / \cdot 21$

$b = \frac{21 \cdot \sin 74^\circ}{\sin 67^\circ}$

$b = 21,93$ cm

2.

$a = 19$ cm

$b = 17$ cm

$\alpha = 21^\circ$

$\gamma = 180 - (18,7 + 21) = 140,3^\circ$

$\frac{b}{a} = \frac{\sin \beta}{\sin \alpha}$

$\frac{17}{19} = \frac{\sin \beta}{\sin 21^\circ}$ $\quad / \cdot \sin 21^\circ$

$\frac{17 \cdot \sin 21^\circ}{19} = \sin \beta$

$18,7^\circ = \beta$

$\frac{c}{b} = \frac{\sin \gamma}{\sin \beta}$

$\frac{c}{17} = \frac{\sin 140,3^\circ}{\sin 18,7^\circ}$ $\quad / \cdot 17$

$c = \frac{17 \cdot \sin 140,3^\circ}{\sin 18,7}$

$c = 33,87$ cm

Hf:

$\alpha = 41^\circ$ $\gamma = 53^\circ$ $b = 13$ cm

$b = 24$ cm $c = 29$ cm $\gamma = 77^\circ$

1. $\alpha = 41^\circ$ $a = 8{,}55$ cm

$\gamma = 53^\circ$ $c = 10{,}41$ cm

$\beta = 86^\circ$ $b = 13$ cm

$\beta = 180^\circ - (41^\circ + 53^\circ) = 86^\circ$

$$\frac{a}{b} = \frac{\sin\alpha}{\sin\beta}$$

$$\frac{a}{13} = \frac{\sin 41^\circ}{\sin 86^\circ} \quad / \cdot 13$$

$$a = \frac{13 \cdot \sin 41^\circ}{\sin 86^\circ}$$

$a = 8{,}55$ cm

$$\frac{c}{b} = \frac{\sin\gamma}{\sin\beta}$$

$$\frac{c}{13} = \frac{\sin 53^\circ}{\sin 86^\circ} \quad / \cdot 13$$

$$c = \frac{13 \cdot \sin 53^\circ}{\sin 86^\circ}$$

$c = 10{,}41$ cm

2. $a = 22{,}55$ cm $\alpha = 49{,}26^\circ$

$b = 24$ cm $\beta = 53{,}74^\circ$

$c = 29$ cm $\gamma = 77^\circ$

$$\frac{b}{c} = \frac{\sin\beta}{\sin\gamma}$$

$$\frac{24}{29} = \frac{\sin\beta}{\sin 77^\circ} \quad / \cdot \sin 77^\circ$$

$$\frac{24 \cdot \sin 77^\circ}{29} = \sin\beta$$

$53{,}74^\circ = \beta$

$$\frac{a}{c} = \frac{\sin\alpha}{\sin\gamma}$$

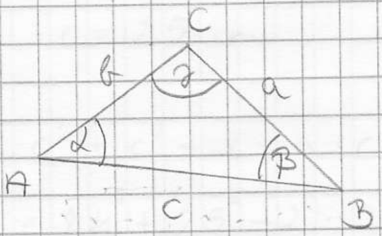
$$\frac{a}{29} = \frac{\sin 49{,}26^\circ}{\sin 77^\circ} \quad / \cdot 29$$

$$a = \frac{29 \cdot \sin 49{,}26^\circ}{\sin 77^\circ}$$

$a = 22{,}55$ cm

$\alpha = 180^\circ - (53{,}74^\circ + 77^\circ) = 49{,}26^\circ$

Koszinusztétel



$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$

$b^2 = a^2 + c^2 - 2 \cdot ac \cdot \cos\beta$

$c^2 = a^2 + b^2 - 2\,ab \cdot \cos\gamma$

Egy háromszögben bármelyik oldal négyzetét megkapjuk, ha a másik két oldal négyzetének összegéből kivonjuk a két oldal és a közbezárt szög koszinuszának kétszeres szorzatát.

- használható, ha adott
  - három oldal
  - két oldal és a közbezárt szög

1, $a = 18$ cm $\alpha = 57{,}12^\circ$

$b = 15$ cm $\beta = 44{,}42^\circ$

$c = 21$ cm $\gamma = 78{,}46^\circ$

$c^2 = a^2 + b^2 - 2ab \cdot \cos\gamma$ $\angle^{a}_{b}$

a, $21^2 = 18^2 + 15^2 - 2 \cdot 18 \cdot 15 \cdot \cos\gamma$

$441 = 324 + 225 - 540 \cdot \cos\gamma$

$441 = 549 - 540 \cdot \cos\gamma$ $/ -549$

$-108 = -540 \cos\gamma$ $/ : (-540)$

$78{,}46^\circ = \gamma$

b, $\dfrac{(21^2 - 18^2 - 15^2)}{(-2 \cdot 18 \cdot 15)} = \cos\gamma$

$\gamma = 78{,}46^\circ$

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$

$18^2 = 15^2 + 21^2 - 2 \cdot 15 \cdot 21 \cdot \cos\alpha$

$\frac{(18^2 - 15^2 - 21^2)}{(-2 \cdot 15 \cdot 21)} = \cos\alpha$

$\alpha = 57,12°$

$\beta = 180° - (57,12° + 78,46°) = 44,42°$

2, $a = 23$ cm $\quad \alpha = 78,87°$

$b = 19$ cm $\quad \beta = 54,13°$

$c = 17,14$ cm $\quad \gamma = 47°$

$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$

$c^2 = 23^2 + 19^2 - 2 \cdot 23 \cdot 19 \cdot \cos 47°$

$c^2 = 293,93 \quad / \sqrt{}$

$c = 17,14$ cm

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$

$23^2 = 19^2 + 17,14^2 - 2 \cdot 19 \cdot 17,14 \cdot \cos\alpha$

$\frac{23^2 - 19^2 - 17,14^2}{-2 \cdot 19 \cdot 17,14} = \cos\alpha$

$78,87° = \alpha$

$\beta = 180° - (47° + 78,87°) = 54,13°$

Hf: $a = 24$ cm $\quad b = 27$ cm $\quad c = 25$ cm

$b = 19$ cm $\quad c = 23$ cm $\quad \alpha = 53°$

1.) $a = 24$ cm $\alpha = 54{,}81^\circ$

$b = 27$ cm $\beta = 66{,}84^\circ$

$c = 25$ cm $\gamma = 58{,}35^\circ$

$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$

$27^2 = 24^2 + 25^2 - 2 \cdot 24 \cdot 25 \cdot \cos\beta$

$\frac{27^2 - 24^2 - 25^2}{-2 \cdot 24 \cdot 25} = \cos\beta$

$66{,}84^\circ = \beta$

$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$

$25^2 = 24^2 + 27^2 - 2 \cdot 24 \cdot 27 \cdot \cos\gamma$

$\frac{25^2 - 24^2 - 27^2}{-2 \cdot 24 \cdot 27} = \cos\gamma$

$58{,}35^\circ = \gamma$

$\alpha = 180^\circ - (58{,}35^\circ + 66{,}84^\circ) = 54{,}81^\circ$

2.) $a = 19{,}08$ cm $\alpha = 53^\circ$

$b = 19$ cm $\beta = 52{,}69^\circ$

$c = 23$ cm $\gamma = 74{,}31^\circ$

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$

$a^2 = 19^2 + 23^2 - 2 \cdot 19 \cdot 23 \cdot \cos 53^\circ$

$a^2 = 364{,}01 \quad \sqrt{\ }$

$a = 19{,}08$ cm

$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$

$23^2 = 19{,}08^2 + 19^2 - 2 \cdot 19{,}08 \cdot 19 \cdot \cos\gamma$

$\frac{23^2 - 19{,}08^2 - 19^2}{-2 \cdot 19{,}08 \cdot 19} = \cos\gamma$

$74{,}31^\circ = \gamma$

$\beta = 180^\circ - (74{,}31^\circ + 53^\circ) = 52{,}69^\circ$

1, $a = 23$ cm $\quad \alpha = 61,25°$

$b = 19$ cm $\quad \beta = 46,4°$

$c = 25$ cm $\quad \gamma = 72,35°$

$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos \gamma$

$25^2 = 23^2 + 19^2 - 2 \cdot 23 \cdot 19 \cdot \cos \gamma$

$\frac{25^2 - 23^2 - 19^2}{-2 \cdot 23 \cdot 19} = \cos \gamma$

$72,35° = \gamma$

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \alpha$

$23^2 = 19^2 + 25^2 - 2 \cdot 19 \cdot 25 \cdot \cos \alpha$

$\frac{23^2 - 19^2 - 25^2}{-2 \cdot 19 \cdot 25} = \cos \alpha$

$61,25° = \alpha$

$\beta = 180° - (61,25° + 72,35°) = 46,4°$

2, $a = 19,37$ cm $\quad \alpha = 52°$

$b = 21$ cm $\quad \beta = 58,67°$

$c = 23$ cm $\quad \gamma = 69,33°$

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \alpha$

$a^2 = 21^2 + 23^2 - 2 \cdot 21 \quad 23 \cdot \cos 52$

$a^2 = 375,27$

$a = 19,37$

$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$

$23^2 = 19{,}37^2 + 21^2 - 2 \cdot 19{,}37 \cdot 21 \cdot \cos\gamma$

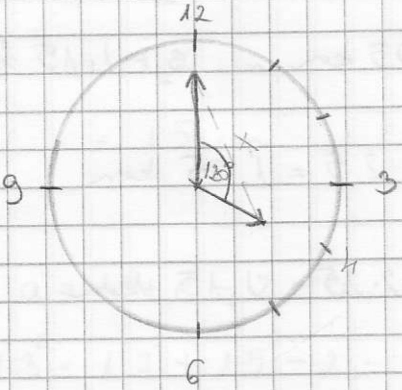
$\frac{23^2 - 19{,}37^2 - 21^2}{-2 \cdot 19{,}37 \cdot 21} = \cos\gamma$

$69{,}33 = \gamma$

$\beta = 180° - (69{,}33° + 52°) = 58{,}67$

3, Egy toronyóra nagymutatója 1,2 m kismutatója 0,7 m.

Mekkora a mutatók végeinek távolsága 16^{00}-kor?



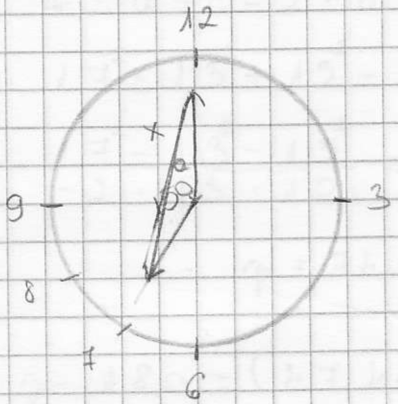
$x^2 = 1{,}2^2 + 0{,}7^2 - 2 \cdot 1{,}2 \cdot 0{,}7 \cdot \cos 120°$

$x^2 = 2{,}77$ $\quad \sqrt{\ }$

$x = 1{,}66$ m

4, Egy karóra nagymutatója 2,3 cm kismutatója 1,8 cm.

Mekkora a mutatók végeinek távolsága 19^{00}-kor?

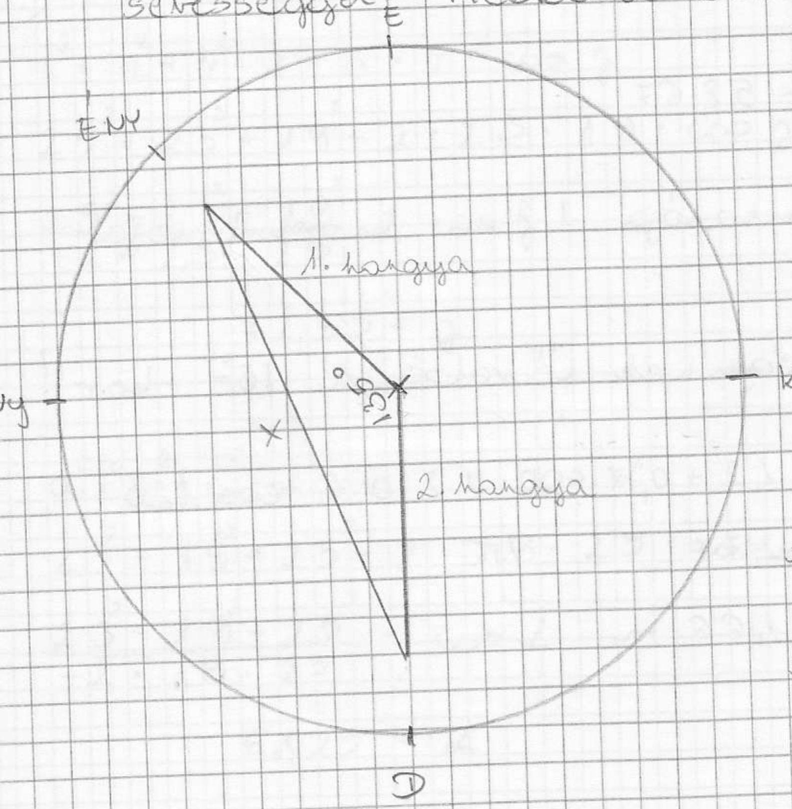


$x^2 = 2{,}3^2 + 1{,}8^2 - 2 \cdot 2{,}3 \cdot 1{,}8 \cdot \cos 150°$

$x^2 = 15{,}7$

$x = 3{,}96$ cm

5, Egy hangya 13.15-kor ÉNY-ra indul a bolyból 0,42 $\frac{km}{h}$ sebességgel. Egy más hangya ugyanonnan 13.30-kor indul D-re 0,51 $\frac{km}{h}$ sebességgel! Mekkora lesz a távolságuk 15.45 kor?



$V = \frac{s}{t} \rightarrow s = v \cdot t$

1. hangya	2. hangya
$V_1 = 0{,}42 \frac{km}{h}$	$V_2 = 0{,}51 \frac{km}{h}$
$t_1 = 2{,}5\ h$	$t_2 = 2{,}25$
$s_1 = 1{,}05\ km$	$s_2 = 1{,}15\ km$

$s_1 = 0{,}42 \cdot 2{,}5 = 1{,}05\ km$

$s_2 = 0{,}51 \cdot 2{,}25 = 1{,}15\ km$

$x^2 = 1{,}05^2 + 1{,}15^2 - 2 \cdot 1{,}05 \cdot 1{,}15 \cdot \cos 135^\circ$

$x^2 = 4{,}13$

$x = 2{,}03\ km$

1,

$a = 17\ cm$

$b = 21\ cm$

$c = 24{,}28\ cm$

$\alpha = 43{,}35^\circ$

$\beta = 58^\circ$

$\gamma = 78{,}65^\circ$

$\frac{a}{b} = \frac{\sin\alpha}{\sin\beta}$

$\frac{17}{21} = \frac{\sin\alpha}{\sin 58^\circ} \quad / \cdot \sin 58^\circ$

$\frac{17 \cdot \sin 58^\circ}{21} = \sin\alpha$

$43{,}35^\circ = \alpha$

$\gamma = 180^\circ - (43{,}35^\circ + 58^\circ) = 78{,}65^\circ$

$$\frac{c}{b} = \frac{\sin\gamma}{\sin\beta}$$

$$\frac{c}{21} = \frac{\sin 78{,}65}{\sin 58^\circ} \quad / \cdot 21$$

$$c = \frac{21 \cdot \sin 78{,}65^\circ}{\sin 58^\circ}$$

$c = 24{,}28$ cm

2, $a = 13$ cm $\qquad \alpha = 47{,}43^\circ$

$b = 17$ cm $\qquad \beta = 74{,}38^\circ$

$c = 15$ cm $\qquad \gamma = 58{,}19^\circ$

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$$

$$13^2 = 17^2 + 15^2 - 2 \cdot 17 \cdot 15 \cdot \cos\alpha$$

$$\frac{13^2 - 17^2 - 15^2}{-2 \cdot 17 \cdot 15} = \cos\alpha$$

$$\alpha = 47{,}43^\circ$$

$$b^2 = a^2 + c^2 - 2 \cdot a \cdot b \cdot \cos\beta$$

$$17^2 = 13^2 + 15^2 - 2 \cdot 13 \cdot 15 \cdot \cos\beta$$

$$\frac{17^2 - 13^2 - 15^2}{-2 \cdot 13 \cdot 15} = \cos\beta$$

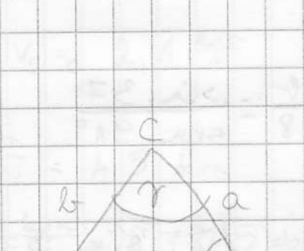
$$\beta = 74{,}38^\circ$$

$\gamma = 180^\circ - (47{,}43^\circ + 74{,}38^\circ) = 58{,}19^\circ$

3, $b = 25$ cm $\qquad \beta =$

$a =$ $\qquad \alpha =$

$c = 21$ cm $\qquad \gamma = 36^\circ$



Egyikkel sem lehet kiszámolni

4. $a = 23\ cm$ $\alpha = 58{,}15^\circ$

$b = 25{,}6\ cm$ $\beta = 71^\circ$

$c = 21\ cm$ $\gamma = 50{,}85^\circ$

$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$

$b^2 = 23^2 + 21^2 - 2 \cdot 23 \cdot 21 \cdot \cos 71^\circ$

$b^2 = 655{,}5$ $/\sqrt{\ }$

$b = 25{,}6$

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$

$23^2 = 25{,}6^2 + 21^2 - 2 \cdot 25{,}6 \cdot 21 \cdot \cos\alpha$

$\frac{23^2 - 25{,}6^2 - 21^2}{-2 \cdot 25{,}6 \cdot 21} = \cos\alpha$

$58{,}15 = \alpha$

$\gamma = 180^\circ - (58{,}15^\circ + 71^\circ) = 50{,}85^\circ$

5. $a = 18\ cm$ $\alpha = 91^\circ$

$b = 10{,}83\ cm$ $\beta = 37^\circ$

$c = 14{,}19\ cm$ $\gamma = 52^\circ$

$\alpha = 180^\circ - (37^\circ + 52^\circ) - 91^\circ$

$\frac{b}{a} = \frac{\sin\beta}{\sin\alpha}$

$\frac{b}{18} = \frac{\sin 37^\circ}{\sin 91^\circ}$ $/ \cdot 18$

$b = \frac{18 \cdot \sin 37^\circ}{\sin 91^\circ}$

$b = 10{,}83\ cm$

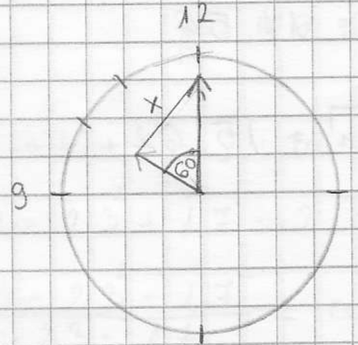
$\frac{c}{a} = \frac{\sin\gamma}{\sin\alpha}$

$\frac{c}{18} = \frac{\sin 52^\circ}{\sin 91^\circ} \quad / \cdot 18$

$c = \frac{18 \cdot \sin 52^\circ}{\sin 91^\circ}$

$c = 14{,}19\ cm$

6, Egy falióra nagymutatója 7,8 cm, kismutatója 5,2 cm
22 órakor mennyi a két mutató végének a távolsága?

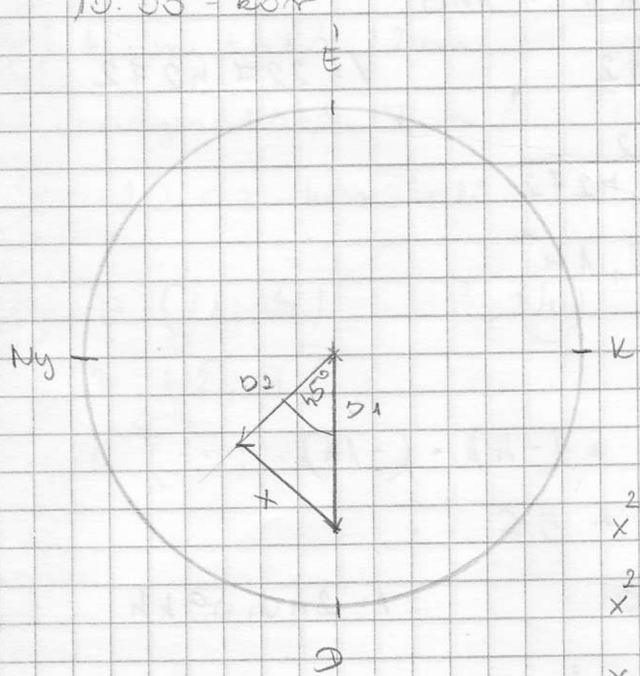


$x^2 = 7{,}8^2 + 5{,}2^2 - 2 \cdot 7{,}8 \cdot 5{,}2 \cdot \cos 60^\circ$

$x^2 = 47{,}32 \quad /\sqrt{\ }$

$x = 6{,}88\ cm$

7, Egy sporthorgász 7,30-kor D-re indul 1,8 km/h sebességgel.
Egy másik ugyanonnan 7,45-kor DNy-ra indul
2,1 km/h sebességgel. Mekkora lesz a távolságuk
10.00-kor



$V = \frac{s}{t} \qquad s = v \cdot t$

$s_1 = 4{,}5\ km \qquad s_2 = 4{,}725\ km$

$t_1 = 2{,}5\ h \qquad t_2 = 2{,}25\ h$

$v_1 = 1{,}8\ \frac{km}{h} \qquad v_2 = 2{,}1\ \frac{km}{h}$

$s_1 = 2{,}5\ h \cdot 1{,}8\ \frac{km}{h} = 4{,}5\ km$

$s_2 = 2{,}22\ h \cdot 2{,}1\ \frac{km}{h} = 4{,}725\ km$

$x^2 = 4{,}725^2 + 4{,}5^2 - 2 \cdot 4{,}725 \cdot 4{,}5 \cdot \cos 45^\circ$

$x^2 = 12{,}51 \quad /\sqrt{\ }$

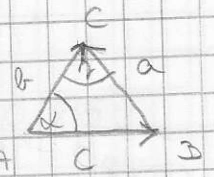
$x = 3{,}54\ km$

2.

$A = (3; 8)$

$B = (-9; -2)$

$C = (7; -6)$



$|\overrightarrow{BC}| = (7-(-9); -6-(-2)) = (16; -4)$

$|\overrightarrow{AC}| = (7-3; -6-8) = (4; -14)$

$|\overrightarrow{AB}| = (-9-3; -2-8) = (-12; -10)$

$a = |\overrightarrow{BC}| = \sqrt{16^2+(-4)^2} = \sqrt{256+16} = \sqrt{272} = 16{,}49$

$b = |\overrightarrow{AC}| = \sqrt{4^2+(-14)^2} = \sqrt{16+196} = \sqrt{212} = 14{,}56$

$c = |\overrightarrow{AB}| = \sqrt{(-12)^2+(-10)^2} = \sqrt{144+100} = \sqrt{244} = 15{,}62$

$a = 16{,}49$ cm $\qquad \alpha = 66{,}14^\circ$

$b = 14{,}56$ cm $\qquad \beta = 53{,}85^\circ$

$c = 15{,}62$ cm $\qquad \gamma = 60{,}01^\circ$

$\alpha = \overrightarrow{AB} \cdot \overrightarrow{AC}$

$15{,}62 \cdot 14{,}56 \cdot \cos\alpha = (-12)\cdot 4 + (-10)\cdot(-14)$

$227{,}4272 \cos\alpha = -48 + 140$

$227{,}4272 \cos\alpha = 92 \qquad / : 227{,}4272$

$\cos\alpha = \dfrac{92}{227{,}4272}$

$\alpha = 66{,}14^\circ$

$\gamma = |\overrightarrow{BC}| \cdot |\overrightarrow{AC}|$

$16{,}49 \cdot 14{,}56 \cdot \cos\gamma = 16\cdot 4 + (-4)\cdot(-14)$

$240{,}0944 \cos\gamma = 64 + 56$

$240{,}0944 \cos\gamma = 120 \qquad / : 240{,}0944$

$\gamma = 60{,}01$

$\beta = 180^\circ - (60{,}01^\circ + 66{,}14^\circ) = 53{,}85^\circ$

3, $a = 19$ cm $\quad\quad \alpha = 54{,}24^\circ$

$b = 23$ cm $\quad\quad \beta = 79{,}2^\circ$

$c = 17$ cm $\quad\quad \gamma = 46{,}56^\circ$

$b^2 = a^2 + c^2 - 2 \cdot a \cdot b \cdot \cos\beta$

$23^2 = 19^2 + 17^2 - 2 \cdot 19 \cdot 17 \cdot \cos\beta$

$\dfrac{23^2 - 19^2 - 17^2}{2 \cdot 19 \cdot 17} = \cos\beta$

$\beta = 79{,}2$

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$

$19^2 = 23^2 + 17^2 - 2 \cdot 23 \cdot 17 \cdot \cos\alpha$

$\dfrac{19^2 - 23^2 - 17^2}{-2 \cdot 23 \cdot 17} = \cos\alpha$

$\alpha = 54{,}24^\circ$

$\gamma = 180^\circ - (54{,}24^\circ + 79{,}2^\circ) = 46{,}56^\circ$

Hf:

1, kismutató 12 cm
nagymutató 17 cm
20.00-kor tali

2, 1. béka ÉNY 14.45-kor $3 \frac{km}{h}$
2. béka K 15.15-kor $2{,}8 \frac{km}{h}$
17.00-kor tali

3, A (11; 3)
B (2; 9)
C (-7; -1)

4, $a = 28$ cm
$b = 21$ cm
$c = 25$ cm

5, $\alpha = 41^\circ$
$\beta = 75^\circ$
$c = 18$ cm